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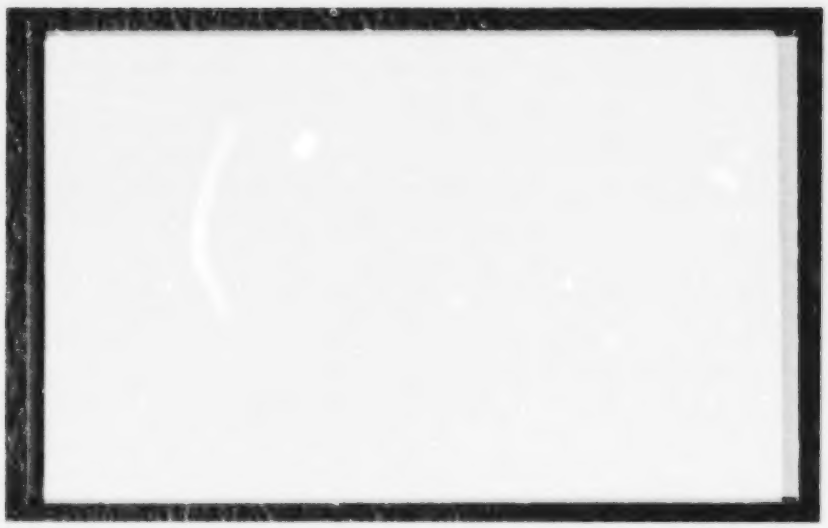

CENTRE DE RECHERCHE SUR LES TRANSPORTS

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MODELS OF SINGLE LANE

TIME HEADWAY DISTRIBUTIONS

BY

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Université de Montréal
Publication #19 - Centre de recherche sur les transports
Mai 1975
Révisé octobre 1975

ABSTRACT

The movement of traffic past a point is likened to the output of a queueing system having random input. A generalisation of the queue output model leads to a suitable headway model: this model is a mixture of two distributions, representing following and non-following headways, in appropriate proportions. This model is compared with several others which have been suggested, and, when used with a lognormal distribution of following headways, it gives the best overall fit to data from the M4 motorway, England, and two-way roads in Indiana, USA. For each site, the parameters of the following headway distribution can be assumed constant. The mean following headways are 1.3 secs and 1.6 secs for the M4 fast and slow lanes respectively, and 2 secs for the Indiana sites. The standard deviation of logarithms of following headways is 0.4 for both M4 lanes and 0.45 for the Indiana sites. For most samples, the reciprocal of the mean inter-bunch gap, λ , can be approximated by $\lambda = \lambda^* - \frac{1}{2} \lambda^{*1\frac{1}{2}}$, where λ^* is the flow rate, and the proportion of following vehicles, ψ , can be approximated by $\psi = \rho - \frac{1}{2} (\rho-1) \lambda^{*1\frac{1}{2}}$, where ρ is the traffic intensity.

Models which have been used to represent single lane time headway distributions are of two main types: simple models consisting of a single statistical distribution which possesses properties of observed headway distributions, and, models which are a mixture of two distributions, representing headways of following and non-following vehicles, in appropriate proportions. The simple models may not yield as great an insight into the underlying factors affecting the arrival pattern, but they usually require fewer parameters to be estimated from data. The effort to estimate the additional parameters of mixed models is justified if they substantially improve goodness-of-fit to data, and if they are related to physically observable quantities.

This paper describes appropriate simple and mixed models and compares their fit to data. The general parameter variations and possible simplifications of the best fitting model are also investigated.

1. SIMPLE MODELS

The lognormal distribution has been found by a number of authors to be the best simple model of the distribution of headways. GREENBERG[1] and TOLLE[2] found that the distribution gave a good fit to headway data obtained at high flows. DAOU[3], HEYES and ASHWORTH[4] and Greenberg found that the distribution gave a good fit to following vehicle headway data obtained from tunnels.

Tolle also found that the introduction of a minimum headway of 0.3 secs improved the fit of the lognormal distribution. BRANSTON[5] has used the maximum likelihood method to estimate the minimum headway parameter for observed headway distributions. However, the estimates were so close to 0.3 secs that he concluded that this was the most suitable value. The introduction of a 0.3 secs minimum headway brought a similar improvement in goodness-of-fit over the standard form to that found by Tolle.

In this paper the lognormal distribution with a minimum headway of 0.3 secs and parameters estimated by the method of maximum likelihood is the only simple model considered.

2. MIXED MODELS

Two mixed models of headways are considered in this paper, a queueing model, and a semi-Poisson model proposed by BUCKLEY[6]. Both models are derived by modifying a random arrival pattern on the assumption that the two physical factors which prevent traffic in a single lane from behaving as a random phenomenon are the constraints imposed by stability and safety considerations, and the constraints imposed by the lack of overtaking opportunities. Safety and stability considerations ensure that following vehicles must assume larger headways than those allowed by a random arrival model. The size of following headways will vary between vehicles and will depend both on the length of the vehicle being followed and the following driver's perception of a safe

separation between vehicles at the speed at which he is driving.

2.1 The generalised queueing model

The modification to Poisson vehicle arrivals imposed by stability and safety considerations is similar to that imposed on a Poisson process by a classical queueing system with a single server. The distribution of following vehicle headways on the road is analogous to the service time distribution in the queueing system. The following vehicle headways are assumed to be independent and have a common distribution. They are also assumed to be independent of the original Poisson arrival pattern.

The distribution resulting from the modification takes the general form for mixed models, i.e.,

$$f(t) = \psi\, g(t) + (1-\psi)\, h(t) \qquad (1)$$

where $f(t)$ is the p.d.f. (probability density function) of all headways,

$g(t)$ is the p.d.f. of following headways,

$h(t)$ is the p.d.f. of non-following headways, and

ψ is the proportion of following vehicles.

Each non-following headway t is the sum of a following headway x drawn from the p.d.f. $g(x)$ and a gap $(t-x)$ which is negative-exponentially distributed with parameter λ^*, the flow rate. From the convolution theorem for the addition of random variables, it follows that

$$h(t) = \int_0^t g(x)\, \lambda^* \exp\,[-\lambda^*(t-x)]\, dx \qquad (2)$$

The proportion of following vehicles is, using queueing theory terminology, the traffic intensity ρ, defined by $\rho = \frac{\text{mean following headway}}{\text{mean headway}}$. Substituting into (1) we then obtain the composite p.d.f. for this queue-output headway model:

$$f(t) = \rho\, g(t) + (1-\rho)\, \lambda^* \exp\,[-\lambda^* t] \int_0^t g(x) \exp\,[\lambda^* x]\, dx \qquad (3)$$

This headway model is analogous to the queueing model of traffic flow proposed by TANNER[7] except that it allows a distribution of following headways rather than a constant following headway. The model allows for stability and safety considerations, but does not allow for overtakings.

The time headway distribution at any downstream point of a section of road which has an input headway distribution given by Eq. (3), will differ from Eq. (3) if the rate at which vehicles overtake on the section of road is not the same as that at which they catch up. We generalise the queue-output headway to take this into account by allowing the proportion of following vehicles and the parameter of the negative-exponential distribution of gaps to vary from the values given in Eq. (3). The composite p.d.f. of headways is then given by

$$f(t) = \psi\, g(t) + (1-\psi)\, \lambda \exp\,[-\lambda t] \int_0^t g(x) \exp\,[\lambda x]\, dx \qquad (4)$$

where $\frac{1}{\lambda}$ is the mean inter-bunch gap.

The generalised queueing model of Eq. (4) is the headway distribution arising from the 'random bunches' model of traffic flow proposed by

MILLER[8]. This type of model was also suggested by BUCKLEY[9] as an alternative to his semi-Poisson model, but, as far as is known to the author, it has not previously been tested against data.

The model is applied in this paper with three different underlying distributions of following headways, the normal, lognormal and gamma distributions.

2.2 Buckley's semi-Poisson model

Buckley's semi-Poisson model of headways differs from the queueing models only in the derivation of the non-following headway distribution. In the semi-Poisson model each non-following headway is obtained by comparing an exponential headway with a following headway, whilst in the queueing models each non-following headway is obtained by adding an exponential gap to a following headway. Buckley showed that the composite p.d.f. for the semi-Poisson model is given by

$$f(t) = \psi\, g(t) + (1-\psi)\, [g^{*}(\lambda)]^{-1}\, \lambda \exp[-\lambda t]\, G(t) \qquad (5)$$

where $g^{*}(\lambda)$ is the Laplace transform (evaluated at λ) of the p.d.f. of following headways, $g(t)$,

$G(t)$ is the distribution function of following headways.

He found that, when applied with a gamma distribution of following headways, the model gave a good fit to data from a freeway in suburban Sydney, Australia.

In this paper, the semi-Poisson model is applied with normal and gamma distributions of following headways. It cannot be applied with a lognormal distribution of following headways, as no explicit expression exists for the Laplace transform of that distribution.

3. THE DATA BASE

In order to test and compare the different headway models fully, time headway data from two different types of site were used. Data were collected from the two-lane overhead section of the M4 motorway in West London by the author using time lapse photography, the procedure being described elsewhere[10]. Data from 50 sites on two-way roads around Bloomington, Indiana were made available by Indiana University. Data from three of these sites were used for testing the headway models.

Two types of sampling procedure were applied to the data; 'short-term', in which data were sampled over short time periods of a minute or so, and a 'long term', procedure due to BREIMAN and LAWRENCE[11], in which data were sampled over time periods (usually tens of minutes) during which average flow was approximately constant. The aim of both these procedures was to eliminate long-term variations of average flow which may produce biased distributions and, consequently, biased estimates of model parameters.

4. USE OF THE CHI-SQUARED TEST OF GOODNESS-OF-FIT

The chi-squared test of goodness-of-fit was used throughout this study to indicate the fit of models to data because of its ease of application to individual samples, and also because it provides a measure of the overall fit of any model to all samples from a site. Classes for use in the test were obtained by dividing the observed data in each sample into ranges with approximately equal frequencies, the number of classes, k, being given by $k = 2.4\ (n-1)^{0.4}$, where ($n \geq 200$) is the sample size, as recommended by KENDALL and STUART[12]. In an attempt to overcome the test's main drawback of producing large chi-squared values which often seemingly contradict good visual fits to large samples, the significance level of the test was allowed to vary with sample size, a value of 5% being used for samples with less than 500 observations and a value of 1% being used for larger samples.

The sum of chi-squared values from all samples gives one measure of the overall fit of a model to data from a site. Unfortunately, this measure may be unduly affected by a few very large individual chi-squared values, and a more generous measure is therefore perhaps more appropriate. The measure used in this paper is a weighted average of the probabilities of obtaining larger values of chi-squared due to random sampling errors in the individual samples. The weight applied to each sample is simply its number of classes. For most sites it was found that if the average probability was less than 10% a model gave a poor fit to at least half the individual samples, and this value can thus be used as a rough estimate for the lower limit of an acceptable overall fit.

5. PARAMETER ESTIMATION FOR MIXED DISTRIBUTIONS

Buckley used the method of moments to estimate parameters of his semi-Poisson model in individual samples. Unfortunately, the method was found to be unsatisfactory for fitting mixed models to the M4 data of this study. It yielded complex parameter estimates in some cases, and the overall goodness-of-fit was very poor. Alternative methods were considered, and the most suitable was found to combine the method of moments with that of minimum chi-squared (the latter method was suggested as an alternative to the method of moments by Buckley). The two parameters of each following headway distribution were allowed to vary whilst parameters λ and ψ were calculated from the first two sample moments. Chi-squared was then minimised using a suitable numerical routine.

The expressions for the required moments of the semi-Poisson model have been derived by Buckley. The general expression for the moments about the origin of the generalised queueing model can be shown to be

$$\mu_{fn} = \mu_{gn} + \frac{n}{\lambda} [\mu_{f(n-1)} - \psi \mu_{g(n-1)}] \qquad (7)$$

where μ_{fn} is the nth moment about the origin of the overall headway distribution,

μ_{gn} is the nth moment about the origin of the following headway distribution,

ψ , λ are as defined previously.

5.1 Comparison of the mixed models

The values of chi-squared obtained by the method of 'minimum chi-squared' from the M4 fast lane short-term sampled data are given in Tables I and II for the semi-Poisson and generalised queueing models, respectively. These tables permit a comparison to be made between the different forms of mixed models.

The overall goodness-of-fit is not acceptable for either model with a normal distribution of following headways. For gamma distributed following headways there is very little to choose between the semi-Poisson and generalised queueing models, both providing an acceptable and almost identical overall goodness-of-fit. However the best overall fit for the fast lane is provided by the generalised queueing model with a lognormal distribution of following headways. Results for the slow lane, which are not shown, followed a similar pattern. Although for the slow lane the lognormal generalised queueing model did not give a substantially better overall fit than either of the mixed models with a gamma distribution of following headways, it did provide an acceptable fit in more individual samples.

The average probability for fast and slow lane results combined was 34.5% for the lognormal generalised queueing model as against 24% for the better gamma case, the semi-Poisson model. This is the first time that the goodness-of-fit of the lognormal generalised queueing to observed data has been reported, and it would seem from the results that, for general application it may be the most suitable of those mixed models tested.

Table I

Results for Buckley's semi-Poisson model fitted by "minimum chi-squared" method

Short-term sampled headway data, M4 fast lane

Flow range (vehs/hr)	Sample size	Following headway distribution: Gamma χ^2	Gamma P	Normal χ^2	Normal P
900-1000	300	24.4	15*	44.7	<1
1000-1100	300	25.3	12*	39.9	<1
1100-1200	700	33.0	23*	49.5	<1
1200-1300	750	30.9	35*	41.1	7
1300-1400	1000	47.7	5*	87.7	<1
1400-1500	700	95.4	<1	145.9	<1
1500-1600	850	40.3	12*	83.3	<1
1600-1700	550	29.7	25*	35.2	8*
1700-1800	200	22.3	10*	29.7	1
1800-1900	500	37.1	4	58.1	<1
2200-2300	250	16.9	45*	19.6	30*
2300-2400	400	18.8	60*	30.2	9*
2400-2500	300	19.8	30*	31.9	2
2500-2600	350	32.1	5*	46.7	<1
2600-2700	200	25.9	4	30.5	1
2900-3000	200	15.2	40*	11.9	70*
	Average probability =		19.6%		6.8%

* Distribution gives an acceptable fit in this range

P is the probability of getting a higher value of chi-squared due to random sampling errors

No. of parameters estimated from data = 4

No. of degrees of freedom = N-5

Table II

Results for generalised queueing model fitted by "minimum chi-squared" method

Short-term sampled headway data, M4 fast lane

Flow Range (vehs/hr	Sample size	Following headway distribution: Gamma χ^2	Gamma P(%)	Lognormal χ^2	Lognormal P(%)	Normal χ^2	Normal P(%)
900-1000	300	24.4	15*	19.1	40*	47.6	<1
1000-1100	300	25.3	12*	19.8	40*	41.1	<1
1100-1200	700	33.0	23*	29.6	40*	50.4	<1
1200-1300	750	30.9	35*	30.8	35*	45.2	3
1300-1400	1000	47.6	5*	31.5	55*	88.7	<1
1400-1500	700	95.4	<1	86.9	<1	151.3	<1
1500-1600	850	40.3	12*	31.1	45*	84.6	<1
1600-1700	550	30.0	22*	29.4	25*	35.2	8*
1700-1800	200	22.3	10*	20.5	17*	30.0	1
1800-1900	500	37.1	4	30.3	18*	58.1	<1
2200-2300	250	16.7	45*	15.2	60*	17.3	40*
2300-2400	400	18.8	60*	15.7	79*	29.3	11*
2400-2500	300	19.9	30*	17.4	50*	30.0	4
2500-2600	350	32.2	5*	30.4	6*	47.3	<1
2600-2700	200	27.6	3	27.1	3	31.8	<1
2900-3000	200	16.4	40*	13.0	60*	16.0	40*
Average probability =			19.3%		35.9%		5.9%

* Distribution gives an acceptable fit in this range

P is the probability of getting a higher value of chi-squared due to random sampling errors

No. of parameters estimated from data = 4

No. of degrees of freedom = N-5

6. COMPARISON OF LOGNORMAL AND LOGNORMAL GENERALISED QUEUEING MODELS

Table III shows the results of the chi-squared analysis for both the 3-parameter lognormal distribution with threshold 0.3 secs and the lognormal generalised queueing distribution fitted to the long-term sampled M4 data. For every sample the lognormal generalised queueing model gives the better fit of the two models, and it is quite clear that in terms of the chi-squared results, it is the better headway model for this data.

Figure 1 compares the visual fit of the two models for the 890 vehs/hr slow lane and 1051 vehs/hr fast lane samples. The plot for the slow lane sample shows that sometimes there will be very little difference between the visual fit of the two models, even though the chi-squared results may indicate otherwise. However, as the fast lane plot shows, there will also be many samples in which the lognormal generalised queueing model gives a much better visual fit to the data. It must therefore be concluded that in terms of both visual and statistical fits the lognormal generalised queueing model is the better headway model for this data.

Figure 2 shows the lognormal generalised queueing model fitted to the M4 slow and fast lane samples with flows of 1772 and 2480 vehs/hr. These samples were collected during the morning peak period when flow on the section of M4 covered was close to capacity. The visual fit of the model to both the slow and fast lane samples appears to be quite good,

Table III

Comparison of goodness-of-fit of 3-parameters lognormal and lognormal generalised queueing models

M4 long-term sampled headway data

Average flow (vehs/hr)	Sample size	Generalised queue χ^2	Generalised queue P(%)	3-parameter lognormal χ^2	3-parameter lognormal P(%)
Slow lane					
890	2238	59.7	10*	89.5	<1
955	1175	48.1	9*	56.2	3*
1018	408	18.6	67*	20.8	65*
1430	297	11.6	86*	35.2	2
1772	1033	50.2	4*	139.4	<1
	Average probability =		28.9%		10.5%
Fast lane					
1051	814	25.9	68*	84.6	<1
1285	707	46.5	2*	87.0	<1
1305	1285	48.9	9*	113.8	<1
1344	964	42.4	11*	105.8	<1
1429	654	30.1	31*	108.0	<1
1551	450	42.6	<1	75.4	<1
1677	921	27.4	70*	91.8	<1
2066	380	29.0	12*	48.7	<1
2480	1944	53.7	18*	69.3	2*
	Average probability =		21.9%		0.7%

* Distribution gives an acceptable fit in this range

P is the probability of getting a higher value of chi-squared due to random sampling errors

No. of degrees of freedom
= N-4 for lognormal distribution
= N-5 for generalised queueing model

and yet for the slow lane sample the probability of obtaining a larger value of chi-squared than that obtained is only 4%. The number of headways in this sample is 1033, and Fig. 2 thus confirms that large samples often produce high values of chi-squared, even though a curve may give a good visual fit to data.

Table IV shows the results of the chi-squared analysis for both the 3-parameter lognormal distribution with threshold 0.3 secs and the lognormal generalised queueing distribution fitted to the long-term sampled Indiana data. As for the M4 data, the overall goodness-of-fit, as measured by the average probability, is better for all three loop sites for the generalised queueing model than for the lognormal distribution. For the M4 data, the generalised queueing model gave the better fit in all individual samples. By contrast, for 3 of the samples from Indiana loop 30, the lognormal distribution gives the better fit. The overall impression remains, however, that the generalised queueing model is better for the description of the Indiana headways.

Figures 3, 4 and 5 show the lognormal generalised queueing model fitted to samples from Indiana loops 5, 10 and 30, respectively. The visual fit of the models to the individual samples generally confirms the chi-squared results obtained, those samples with a high value of P, the probability of having a larger chi-squared value, having a good visual fit and *vice versa*. However, even for those samples having the lowest values of P the visual fit of the lognormal generalised queueing model seems reasonable.

Table IV

Comparison of goodness-of-fit of 3-parameter lognormal and lognormal generalised queueing models

Indiana long-term sampled headway data

Average flow (vehs/hr)	Sample size	Generalised queue χ^2	Generalised queue P(%)	3-parameter lognormal χ^2	3-parameter lognormal P(%)
Loop 5					
148	370	28.9	12*	98.8	<1
153	200	50.2	<1	87.0	<1
223	200	24.3	6*	33.2	1
	Average probability =		6.7%		0.5%
Loop 10					
48	200	19.3	20*	38.8	<1
270	220	22.1	11*	47.9	<1*
281	200	16.2	38*	21.3	21
337	200	25.3	5*	46.4	<1
362	354	20.2	45*	44.3	<1
369	397	31.8	5*	60.6	<1*
434	362	14.7	79*	22.9	41
462	386	34.8	3*	50.7	<1
482	290	20.8	29*	32.2	4
535	372	20.1	52*	44.4	<1*
552	247	14.3	65*	27.7	9
	Average probability =		32.7%		7.2%
Loop 30					
236	200	18.5	14*	23.5	14*
237	272	24.1	16*	48.9	<1
263	200	29.1	2	33.5	1
300	201	41.5	<1*	41.7	<1
313	213	19.5	20*	32.1	2
325	265	19.7	29*	31.6	4
590	253	19.4	31*	37.6	<1
593	348	35.8	2*	52.1	<1*
767	200	14.1	52*	15.9	53
808	319	24.3	19*	49.5	<1*
902	206	24.1	7*	22.6	17*
1006	252	22.8	16*	22.7	25*
	Average probability =		17.2%		9.4%

* Distribution gives an acceptable fit in this range

P is the probability of getting a higher value of chi-squared due to random sampling errors

No. of degrees of freedom
= N-4 for lognormal distribution
= N-5 for generalised queueing model

The visual fits and chi-squared results thus indicate that the lognormal generalised queueing model realistically describes the Indiana headways. This is very encouraging as all the sites are at the ends of sections of road with no overtaking. COWAN[13] has shown theoretically that, for such roads, if the gap between queues is negative-exponentially distributed at the start of the no overtaking section, it will not be so at the end. The generalised queueing model assumes negative-exponentially distributed gaps between queues. The results from the Indiana sites hence show that the assumption is at least a good approximation, even for no-overtaking sections. Taking the results from the M4 and Indiana sites together it is clear that the previously untested lognormal generalised queueing model gives a very good representation of observed headway data at a very wide range of flows and site conditions.

7. PARAMETER VARIATION FOR THE LOGNORMAL GENERALISED QUEUEING MODEL

In discussing the variation of parameter estimates of the lognormal generalised queueing model obtained in this study, only results from long-term samples for which the model gave an acceptable fit are considered. Samples with mean following headways greater than 3 secs are excluded.

Observed variations in the proportion of following vehicles, ψ , and the reciprocal of the mean gap between bunches, λ , are perhaps best expressed in terms of the values which these parameters would take in the queue-output headway model, namely the traffic intensity, ρ ,

and the flow rate λ^* respectively. In the large majority of samples the 'minimum chi-squared' estimate of ψ was greater than ρ, and λ was less than λ^* at all flows. There were also several samples in which ψ was less than ρ and λ greater than λ^*, half of these coming from the M4 slow lane where all samples at flows less than about 1100 vehs/hr exhibited this property. For comparative purposes direct estimates of ψ were obtained for all such samples by classifying vehicles into following and non-following groups using suitable time headway and relative speed criteria. Table V compares these direct estimates with the minimum chi-squared estimates.

Table V

Comparison of direct and 'minimum chi-squared' estimates of ψ

Site	Flow (vehs/hr)	Estimates of ψ	
		Direct	Minimum chi-squared
M4 slow lane	890	0.44	0.10
	955	0.48	0.37
	1018	0.53	0.46
M4 fast lane	2480	0.86	0.82
Indiana loop 30	808	0.57	0.38
	1006	0.57	0.47

For every sample the directly estimated ψ is greater than the 'minimum chi-squared' estimate, implying that the latter method underestimates the true proportion of queueing vehicles in these samples. It may thus be expected that for these samples the goodness-of-fit would be improved by

using a direct method of parameter estimation. However, it was found that in fact the goodness-of-fit was drastically reduced using this method. One possible explanation is that originally suggested by Buckley, that under some traffic conditions these are in fact two distinct types of following driver with different p.d.f.'s of following headways. Under such circumstances, any simple mixed model of the type considered here may then give the best fit to data if it represents only one of the two types of queueing drivers as being 'followers'.

Figure 6 shows plots of parameters μ and σ, the mean and variance respectively of the natural logarithms of following headways, versus flow for the M4 data. The parameters tend to decrease as flow increases, and as the mean following headway is $\exp(\mu + \frac{1}{2}\sigma^2)$, it will also tend to decrease as flow increases. Unfortunately the scatter of μ and σ for the Indiana data was such that it was difficult to draw any concrete conclusions for their variation with flow, and the results are not reproduced here.

Figure 7 shows plots of parameters μ and σ versus speed for several sites. The values were obtained by dividing the directly determined following vehicle data into 5ft/sec speed ranges and calculating maximum likelihood estimates of μ and σ for each range. Data from the M4 slow lane at flows less than 1100 vehs/hr and Indiana loop 30 were excluded because of the possibility of there being two types of following driver present. The contrast between the variability of μ and that of σ between sites is quite striking, μ varying from 0.075

to 0.75 whilst σ remains relatively constant in the range 0.28 to 0.5. The mean value $\sigma = 0.4$ from the combined M4 slow and fast lane values is consistent with the mean 'minimum chi-squared' estimates shown in Fig. 6.

Both μ and σ increase as speed increases at all sites. This result is contrary to that found by Daou that for data from the Holland tunnel, New York, σ was approximately constant with speed. However, all Daou's results were for speeds less than 50ft/sec, whereas the vast majority of results in Fig. 7 are for speeds greater than 50ft/sec. The increase of μ and σ with increasing speed partly explains why in Fig. 6, both μ and σ were found to decrease as flow increased for the M4 data. This was probably because the average speed of traffic decreased as flow increased.

8. A SIMPLIFYING ASSUMPTION

The most obvious simplification of the lognormal generalised queueing model is to assume that parameters μ and σ remain constant. The constant values of μ and σ used to test the validity of this assumption were the arithmetic means of the 'minimum chi-squared' estimates which are shown in Table VI.

Table VI

Mean minimum chi-squared estimates of μ and σ and corresponding mean following headways

Site	Arithmetic Mean μ	Arithmetic Mean σ	Mean Following Headway (secs)
M4 slow lane	0.39	0.42	1.62
M4 fast lane	0.17	0.41	1.29
Indiana loop 5	0.55	0.46	1.93
Indiana loop 10	0.58	0.47	1.99
Indiana loop 30	0.64	0.44	2.09

New estimates of λ and ψ which minimised chi-squared with these constant values of μ and σ were calculated. Table VII shows the average probabilities of obtaining larger chi-squared values due to random sampling errors with the simplifying assumption. For the M4 data the average probabilities compare very favourably with values obtained by the method of 'minimum chi-squared'. For Indiana loops 5 and 30, the overall fits were marginally reduced by the simplification. For loop 10, although the overall goodness-of-fit was apparently reduced considerably by assuming constant μ and σ, the model still provides an acceptable fit.

Table VII

Average probabilities when μ and σ are held constant

	M4 lane		Indiana loop		
	Slow	Fast	5	10	30
Average Probability (%)	25.0	24.2	5.8	23.3	17.0

Figure 8 compares the visual fit of the constant μ and σ and original 'minimum chi-squared' versions of the model for two low flow Indiana samples, the 223 vehs/hr sample of loop 5 and the 48 vehs/hr sample of loop 10. The versions with constant μ and σ seem to give a better visual fit to the data around the major peaks than do the "minimum chi-squared' versions. For the loop 10 sample, the simplification also reduced the value of ψ to 0.1 and the estimated mean following headway to 2 secs from the previous unrealistically high values of 0.29 and 17 secs respectively.

The general conclusion is that an assumption of constant μ and σ is an acceptable simplification in fitting the lognormal generalised queueing model to headway data. Further from the good visual fit to the loop 10 sample of Fig. 8, it can be concluded that an assumption of cons-

tant μ and σ values may overcome difficulties of parameter estimation at very low flows.

As pointed out by Buckley and more recently by WASIELEWSKI[14], the reciprocals of mean following headways predicted by a mixed model give estimates of the 'possible capacity' of a lane. Wasielewski fitted Buckley's semi-Poisson model to data from the M4 motorway and obtained possible capacity estimates of 2230 vehs/hr for the slow lane and 2650 vehs/hr for the fast lane with 13.5% and 1.5% trucks respectively. The mean following headways of Table VI yield possible capacities of 2440 vehs/hr for the M4 slow lane and 2960 vehs/hr for the fast lane with 16.6% and 1.7% trucks respectively. The latter is virtually identical to the highest observed hourly flow of 2930 vehs/hr in the fast lane of a two lane section of the M4 reported by CAPLAN and DUNCAN[15]. However, it is apparent from the fairly large discrepancies in predicted values that it may be misleading to calculate possible capacities from the results of just one sample.

9. PARAMETER ESTIMATION FOR SIMULATION STUDIES

The main drawback of any mixed model is that four parameter estimates must be obtained before the model can be applied. There is thus an obvious need for some prior knowledge of suitable parameter values. In this study the lognormal generalised queueing model of headways has probably been tested against a wider range of data than any previously suggested model. The range of flows is approximately 50 vehs/hr to 2500 vehs/hr, whilst the two sites studied provide quite different qualities of service, the M4 being a relatively high class expressway and the Indiana sites being fairly low class two-way roads. The results

of this paper thus yield some useful general conclusions on values of the lognormal generalised queueing model parameters which may be suitable for simulation studies in the absence of any other information.

Perhaps the main result is that parameters μ and σ can be held constant without a great reduction in goodness-of-fit. The constant σ values varied only by about ± 5% between sites, values of 0.4 and 0.45 being good approximations for the M4 and Indiana sites respectively. As a first approximation it may be possible to classify a site as providing good or poor quality service and to assume 0.4 or 0.45 respectively as an appropriate value for σ. When a suitable value of σ has been chosen, μ can be determined from the mean following headway, α, since $\mu = \ell n\, \alpha - \frac{1}{2}\sigma^2$. A suitable mean following headway could be selected from the predicted values of this paper, which were approximately, 1.3 secs and 1.6 secs for the M4 fast and slow lanes respectively, and 2 secs for the Indiana sites.

For samples with λ less than λ^*, estimates of λ obtained with constant μ and σ values were approximately related to λ^* by

$$\lambda = \lambda^* - \tfrac{1}{2}\lambda^{*1\frac{1}{2}}$$

and ψ values were approximately related to λ^* by

$$\psi = \rho - \tfrac{1}{2}(\rho-1)\lambda^{*1\frac{1}{2}} .$$

Figures 9 and 10 show these 'power' curves fitted to the relevant λ and ψ estimates. The mean following headways used to calculate ρ were 2 secs for the Indiana data, and 1.3 secs for the M4 fast lane samples (the only slow lane sample is excluded). It is apparent that

the power curves give good λ and ψ estimates at all flows for both sets of data. Unfortunately, insufficient samples were available to estimate the parameters of any curves in cases when λ was greater than λ^*.

ACKNOWLEDGEMENT

It gives me very great pleasure to thank all those people who have helped me in the course of this study. I am particularly grateful to J.G. WARDROP, who supervised the work, and to D.J. BUCKLEY and R.E. ALLSOP for invaluable advice and encouragement. I am indebted to K. JOSCELYN of Indiana University for the provision of data, and to the Science Research Council for their overall financial support.

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FIGURE LEGEND

Fig. 1 - Lognormal generalised queueing model and 3-parameter lognormal distributions fitted to M4 headway data.

Fig. 2 - Lognormal generalised queueing model fitted to M4 headway data.

Fig. 3 - Lognormal generalised queueing model fitted to Indiana loop 5 headway data.

Fig. 4 - Lognormal generalised queueing model fitted to Indiana loop 10 headway data.

Fig. 5 - Lognormal generalised queueing model fitted to Indiana loop 30 headway data.

Fig. 6 - Plots of parameters μ and σ of the lognormal following headway distribution versus flow, M4 data.

Fig. 7 - Plots of parameters μ and σ versus speed.

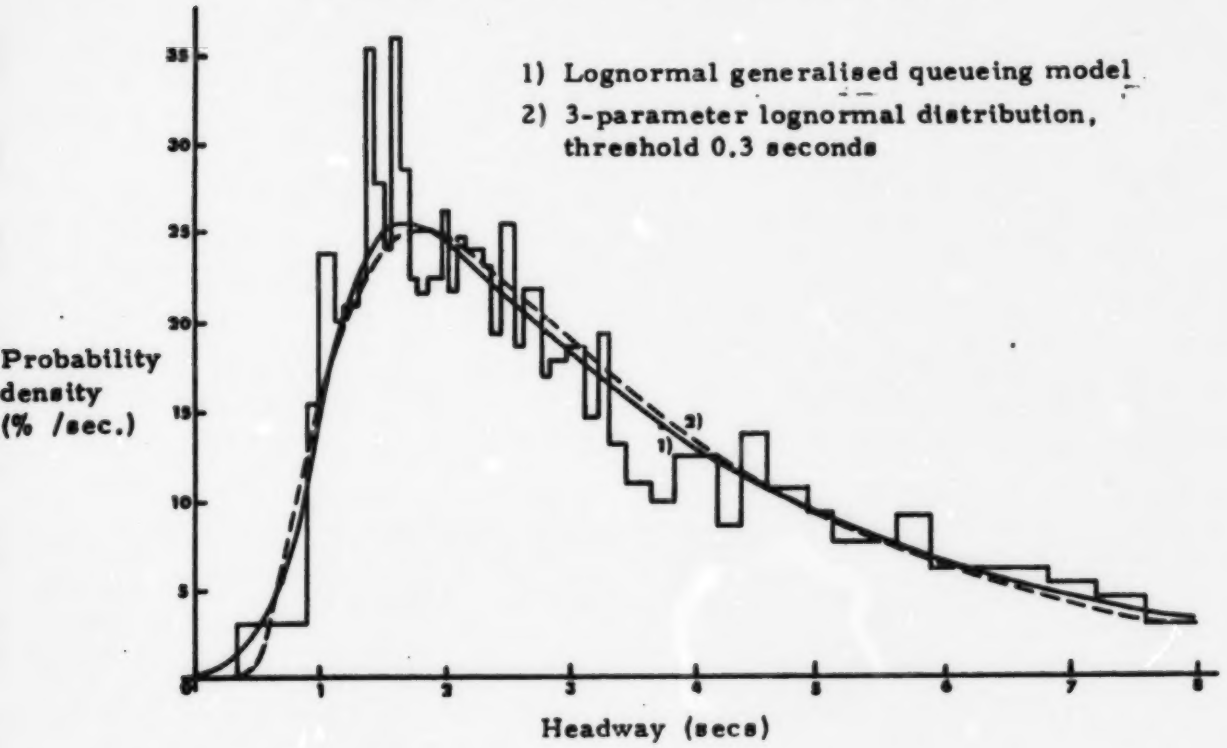
Fig. 8 - Lognormal generalised queueing model with constant μ and σ values fitted to Indiana headway data.

Fig. 9 - 'Power' curves fitted to λ , reciprocal of mear inter-bunch gap, versus λ^* , flow rate, Indiana and M4 data.

Fig. 10- 'Power' curves fitted to ψ , proportion of following vehicles, versus λ^* , flow rate, Indiana and M4 data.

figure 1

M4 SLOW LANE, Average Flow 890 VEHS/HR


1) Lognormal generalised queueing model
2) 3-parameter lognormal distribution, threshold 0.3 seconds
Probability density (% /sec.)
Headway (secs)

M4 FAST LANE, Average Flow 1051 VEHS/HR

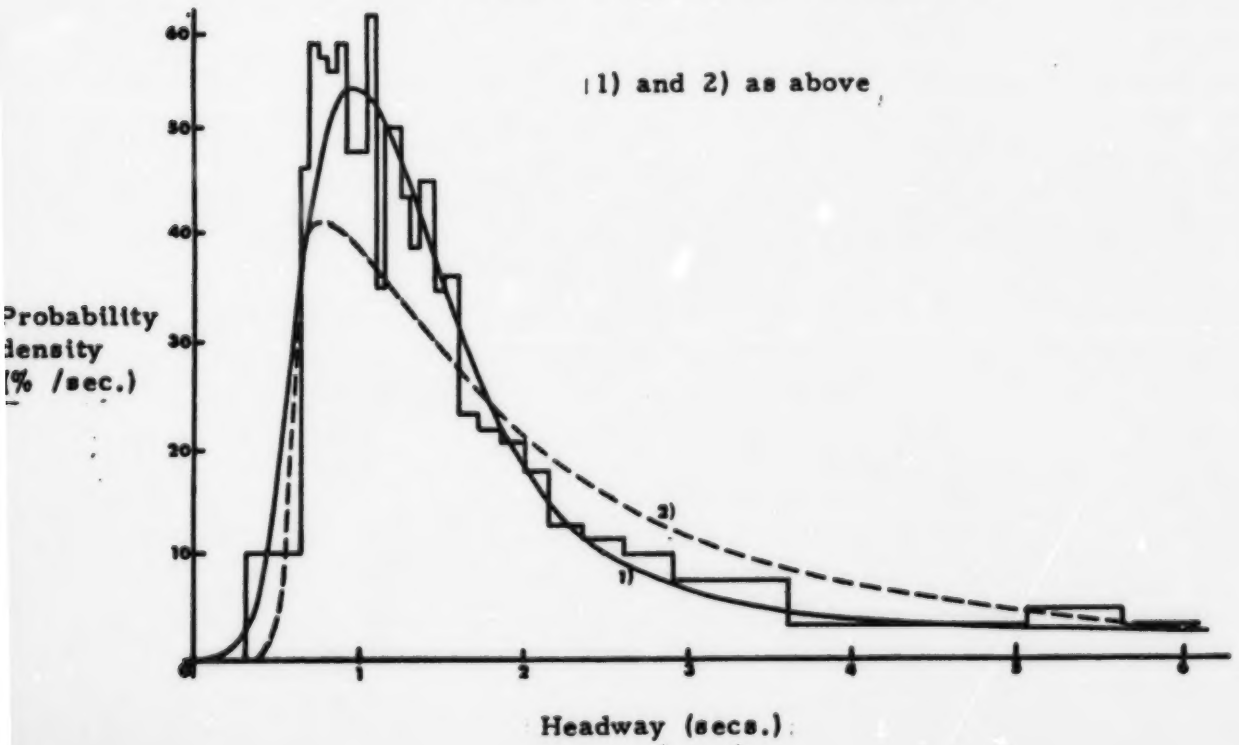
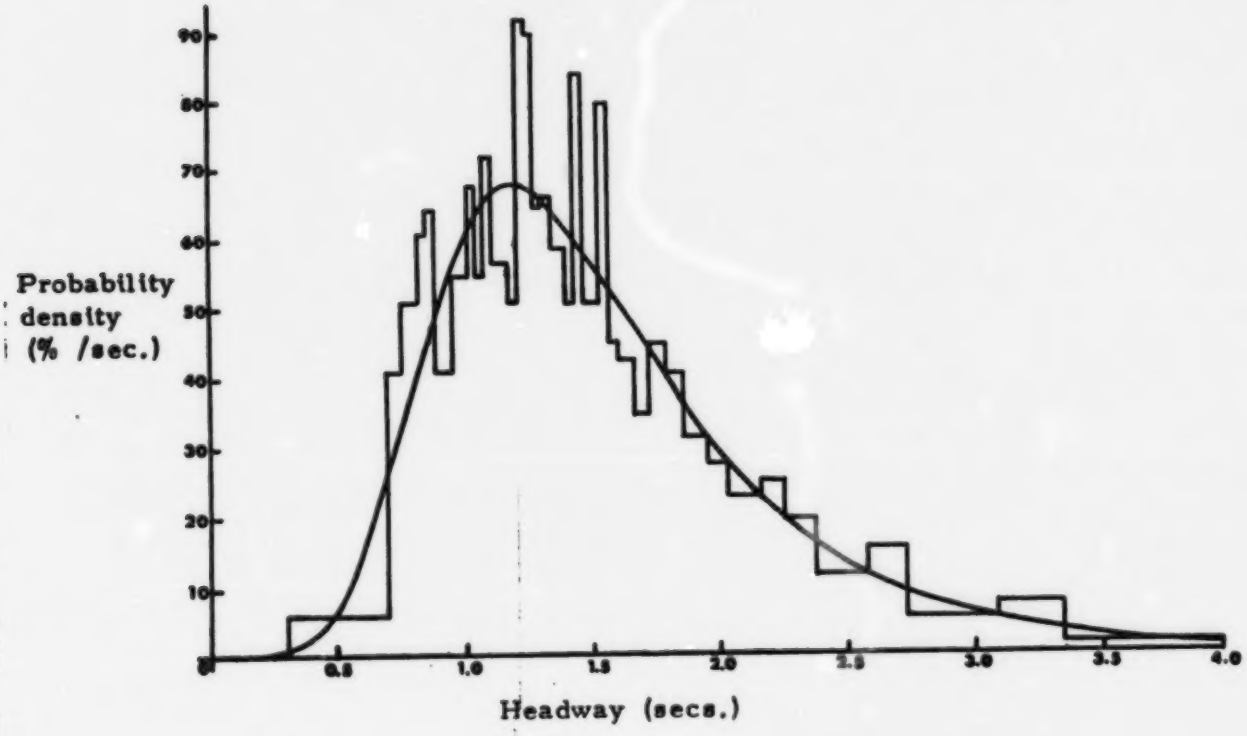


figure 2

M4 SLOW LANE : Average Flow 1772 VEHS/HR



M4 FAST LANE, Average Flow 2480 VEHS/HR

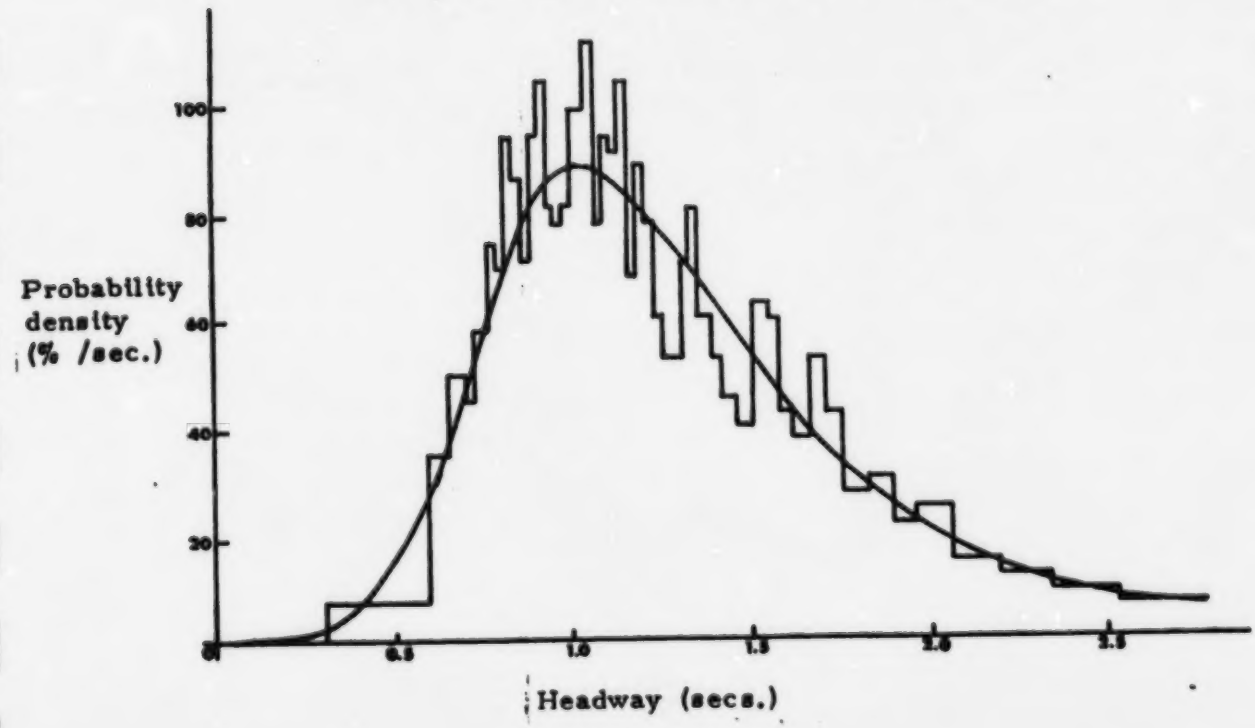


figure 3

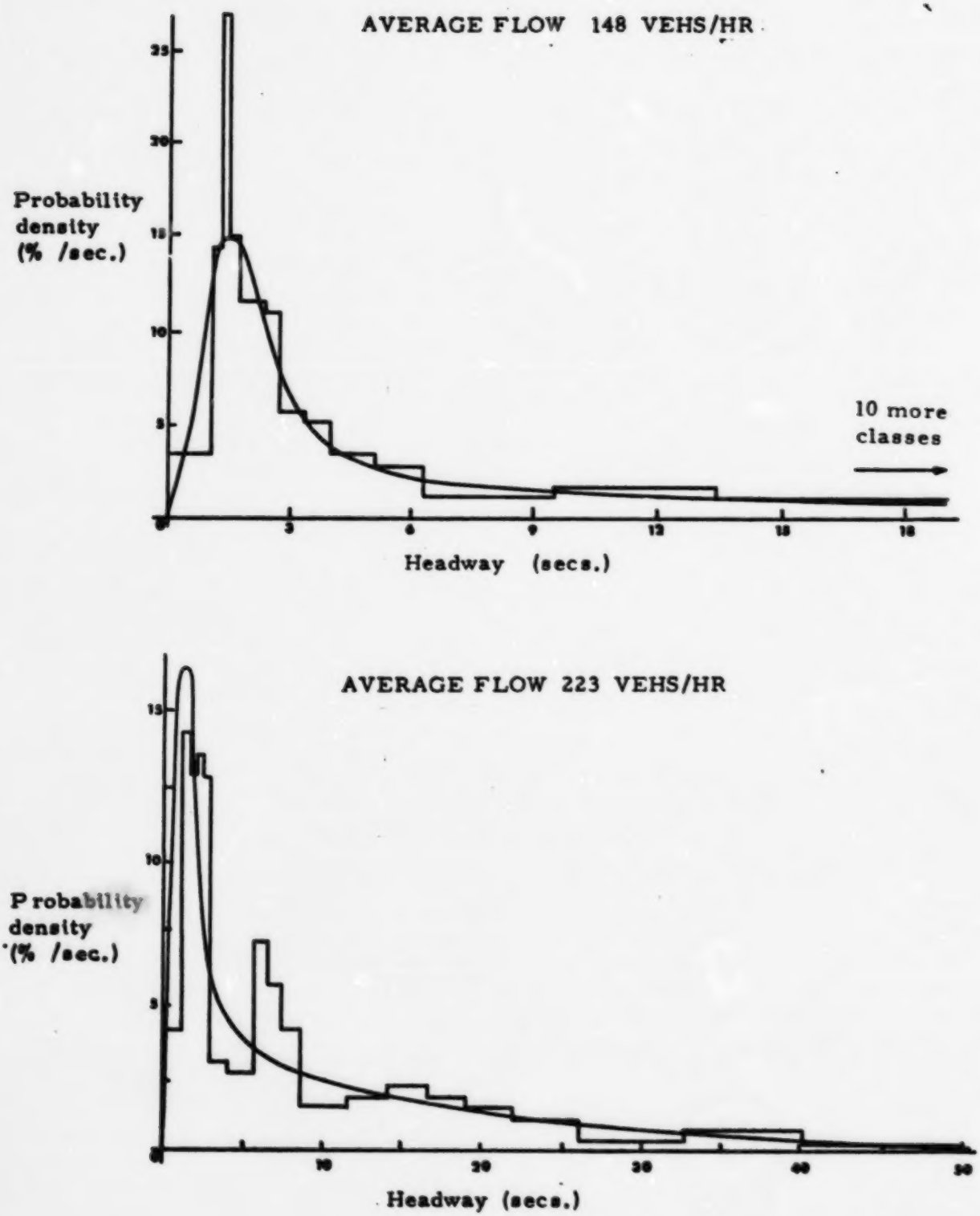
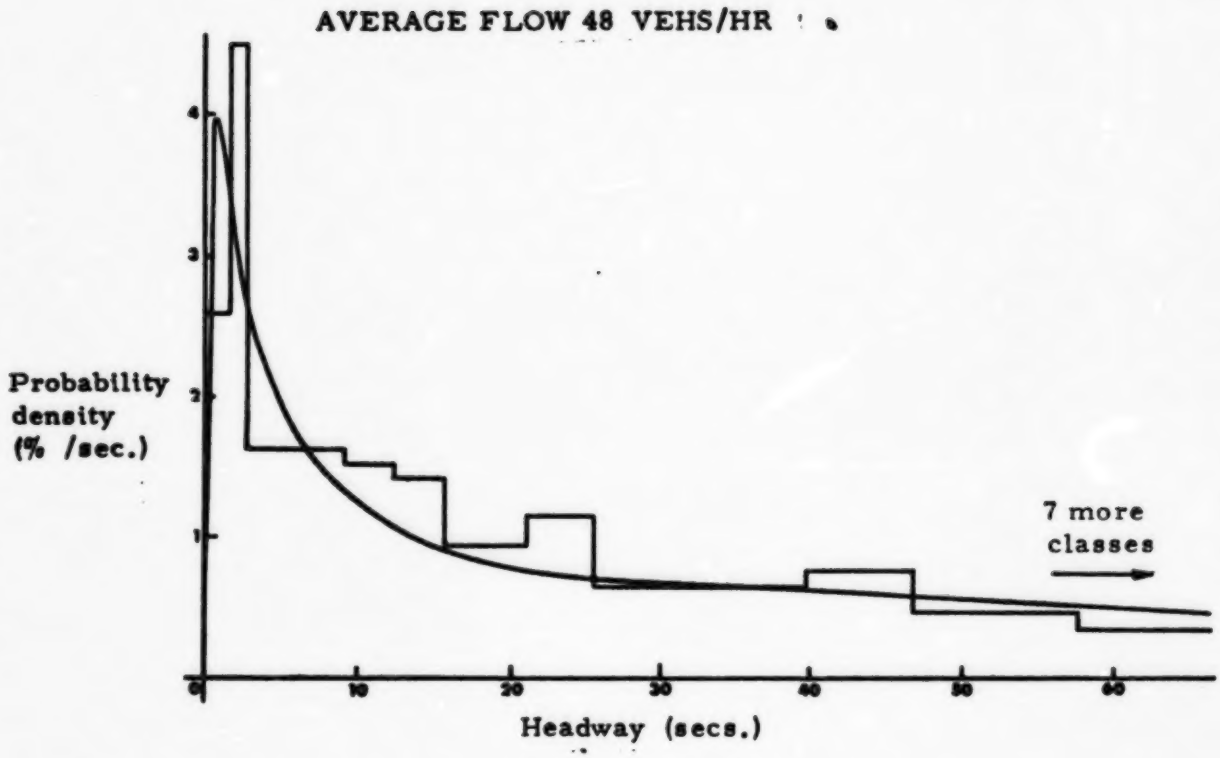

AVERAGE FLOW 148 VEHS/HR
Probability density (% /sec.)
25
20
15
10
5
0
3
6
9
12
15
18
10 more classes
Headway (secs.)
AVERAGE FLOW 223 VEHS/HR
Probability density (% /sec.)
15
10
5
0
10
20
30
40
50
Headway (secs.)

figure 4


AVERAGE FLOW 48 VEHS/HR
Probability density (% /sec.)
7 more classes
Headway (secs.)

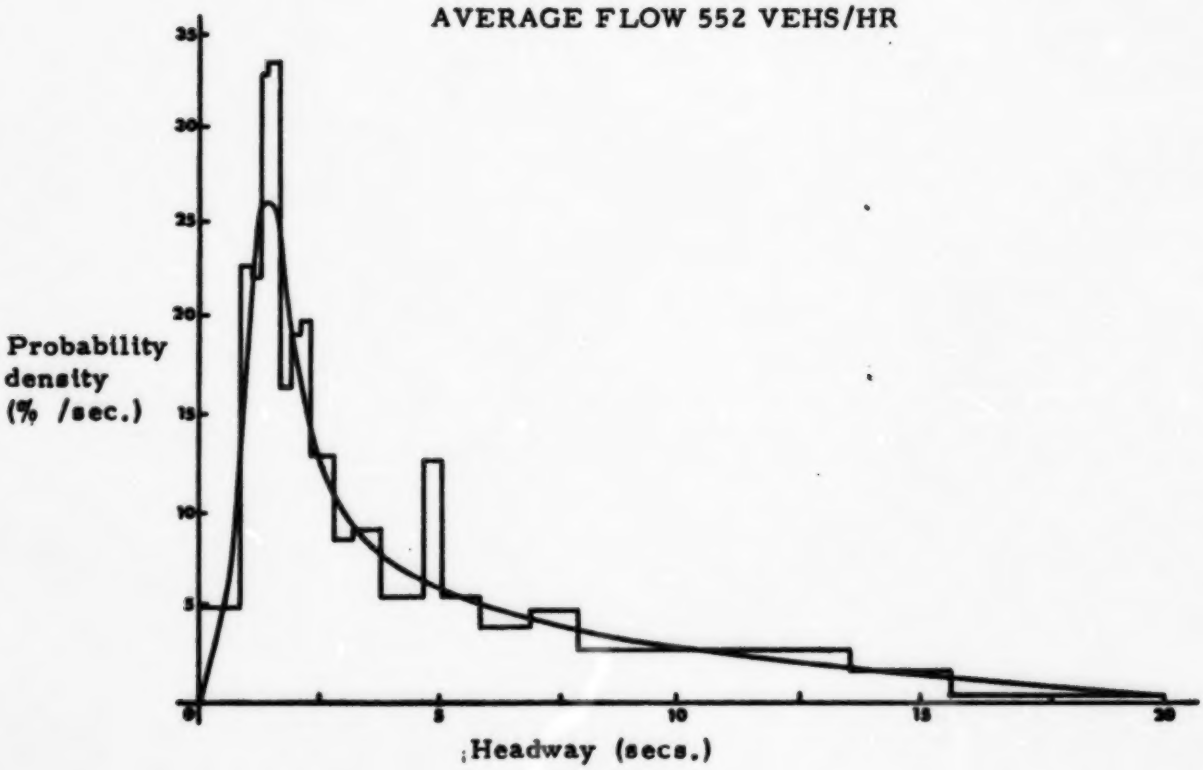

AVERAGE FLOW 552 VEHS/HR
Probability density (% /sec.)
Headway (secs.)

figure 5

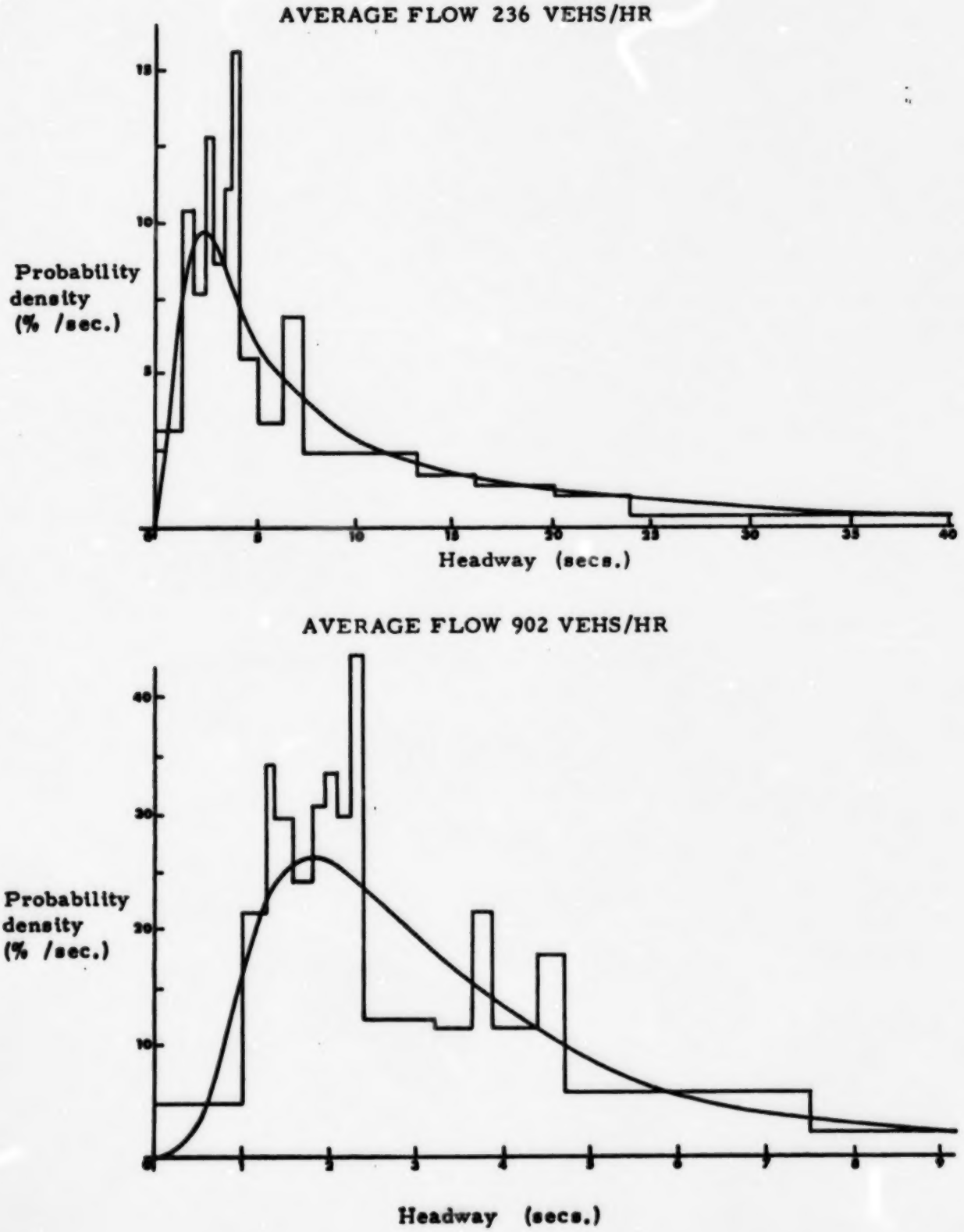
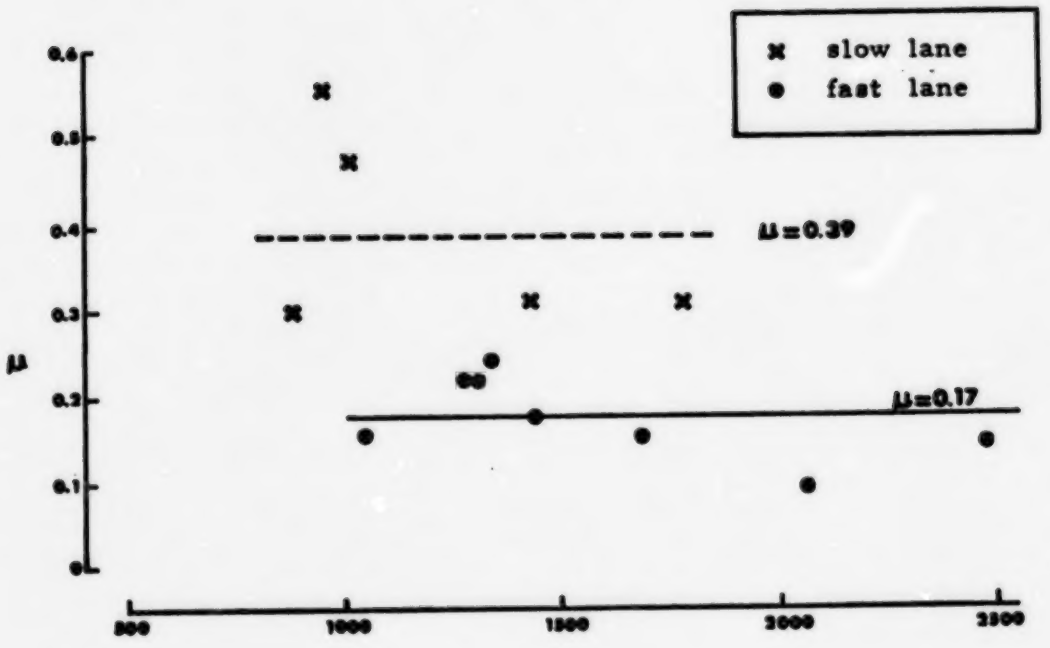
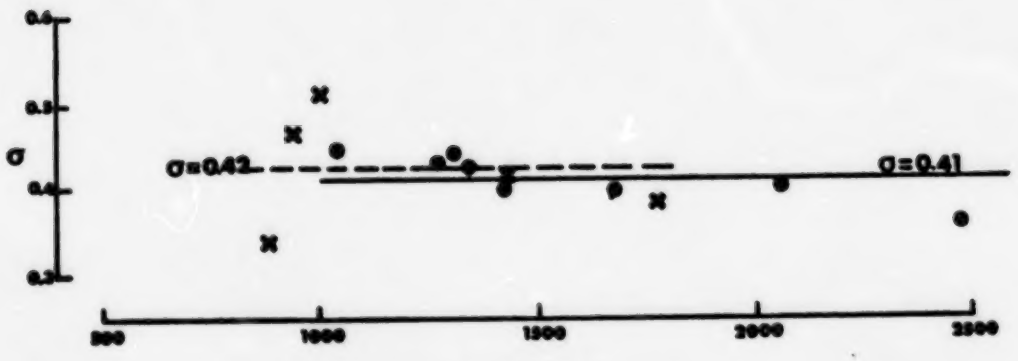

AVERAGE FLOW 236 VEHS/HR
Probability density (% /sec.)
Headway (secs.)
AVERAGE FLOW 902 VEHS/HR
Probability density (% /sec.)
Headway (secs.)

figure 6

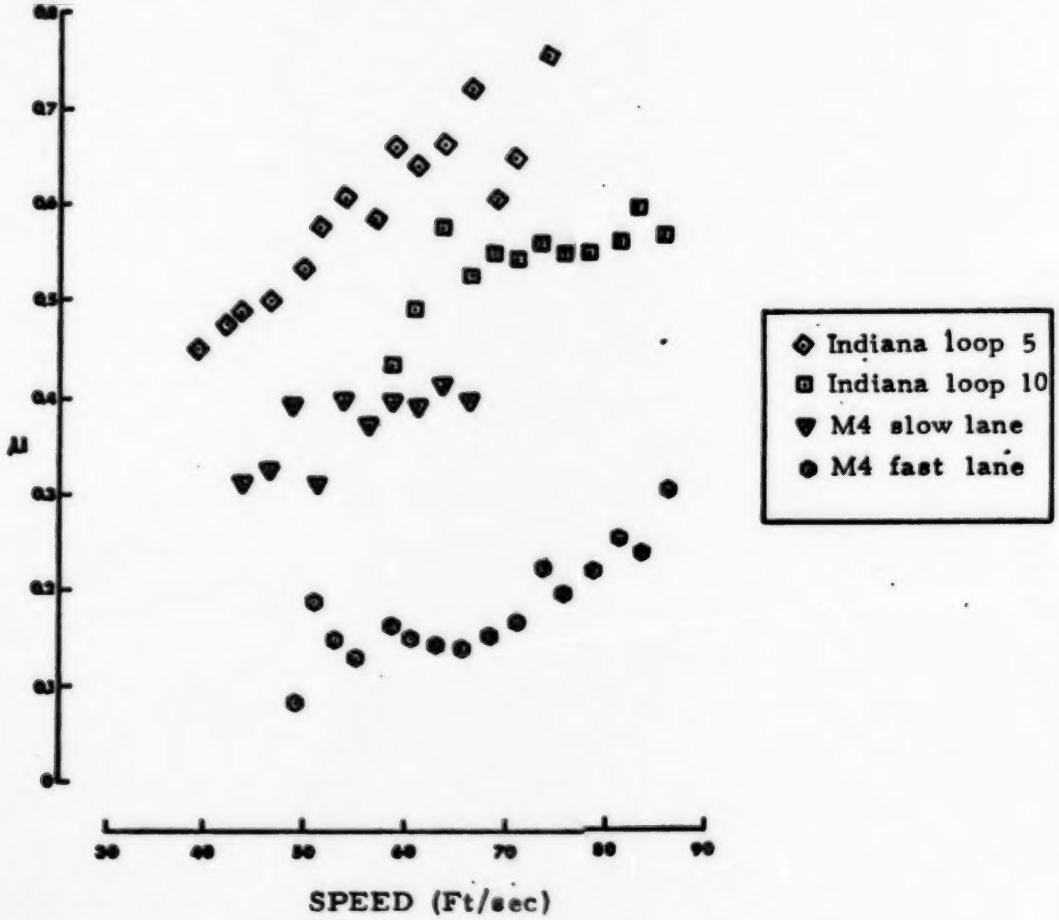


FLOW (vehs/hr)



FLOW (vehs/hr)

figure 7


μ
SPEED (Ft/sec)
Indiana loop 5
Indiana loop 10
M4 slow lane
M4 fast lane

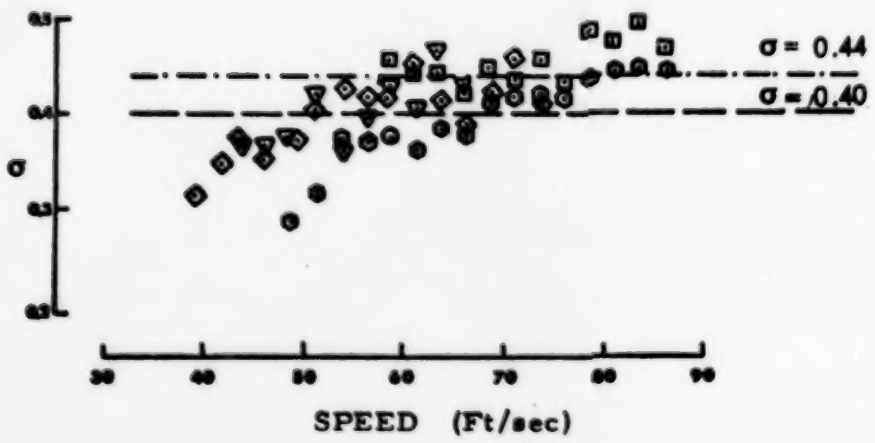
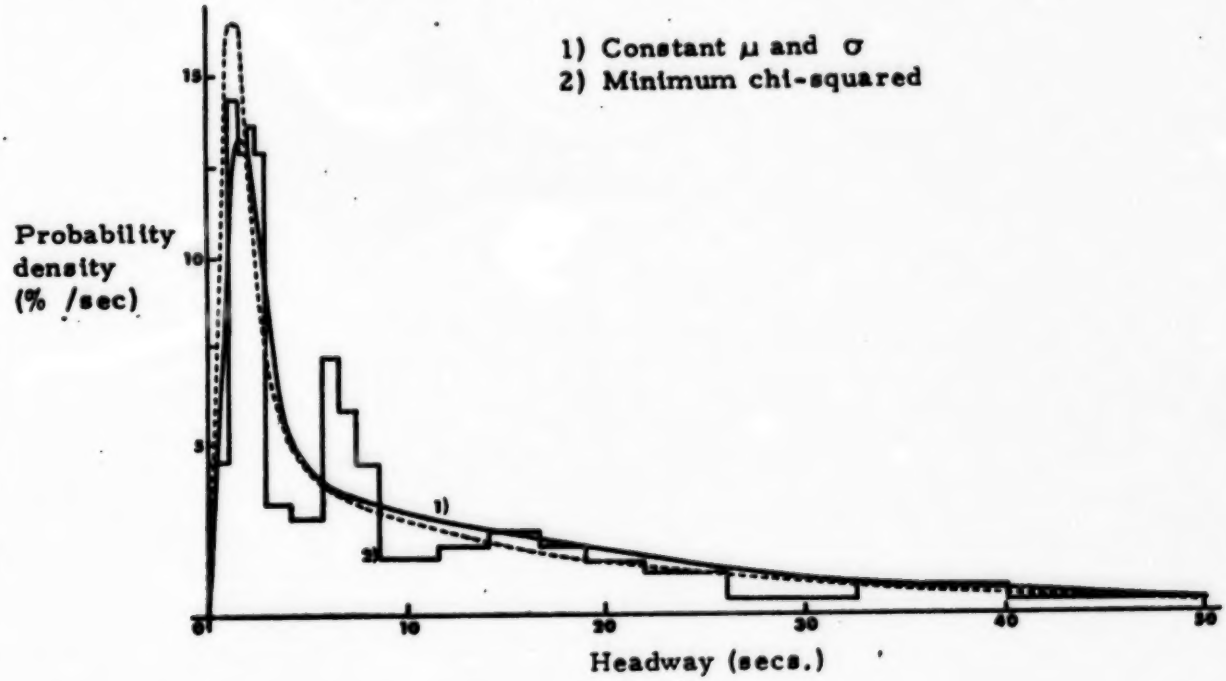

σ
σ = 0.44
σ = 0.40
SPEED (Ft/sec)

figure 8

INDIANA LOOP 5, Average Flow 223 VEHS/HR

1) Constant μ and σ
2) Minimum chi-squared



INDIANA LOOP 10, Average Flow 48 VEHS/HR

1) and 2) as above

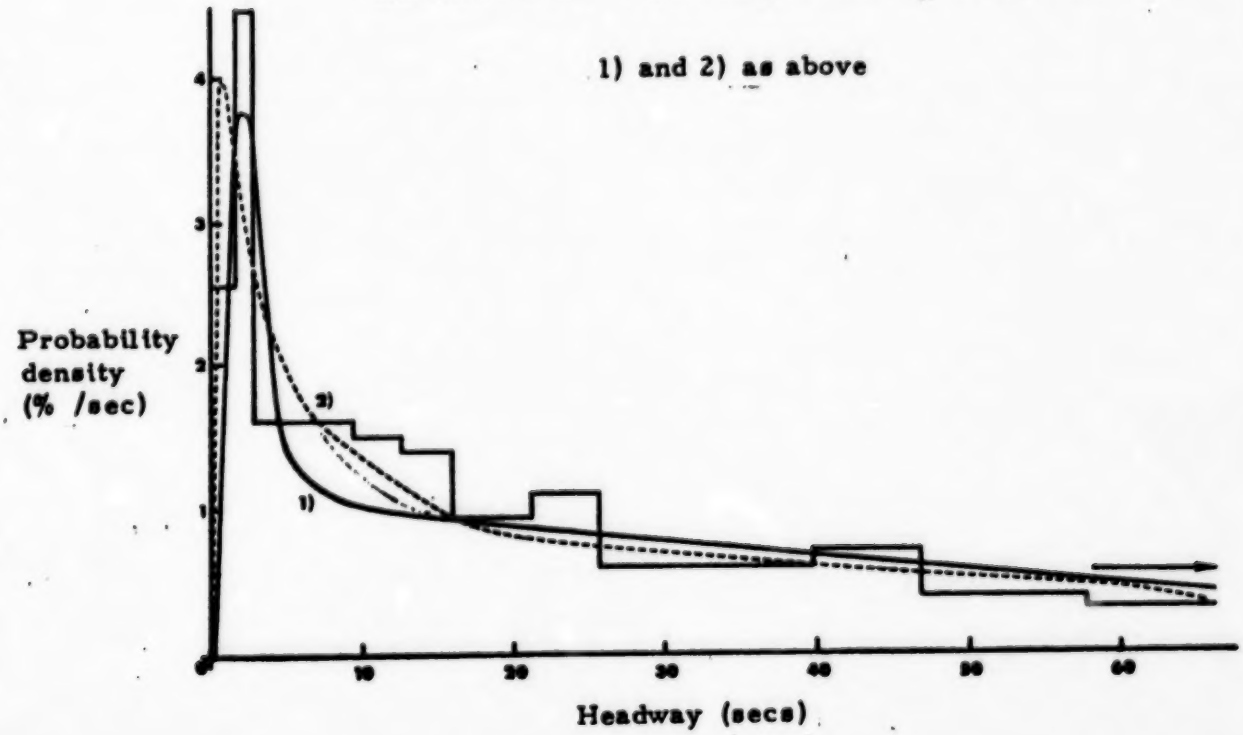
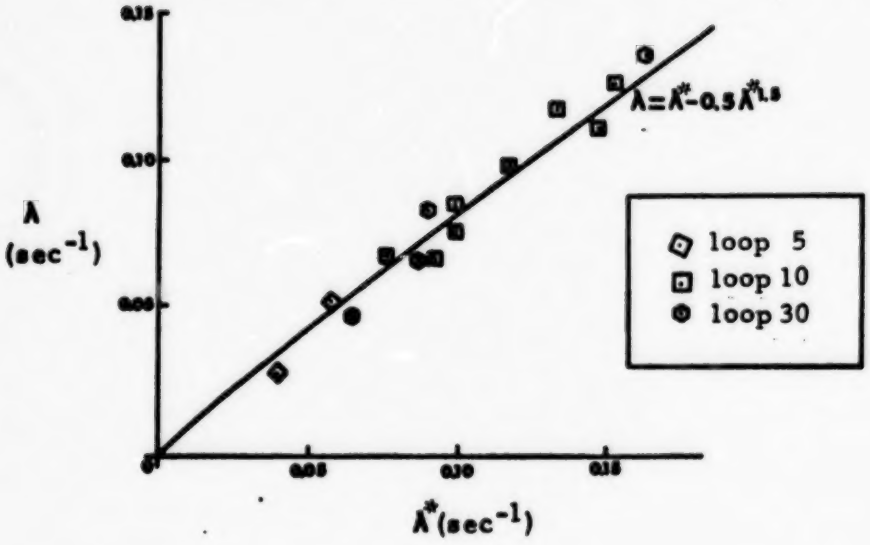


figure 9

INDIANA DATA .



M4 DATA

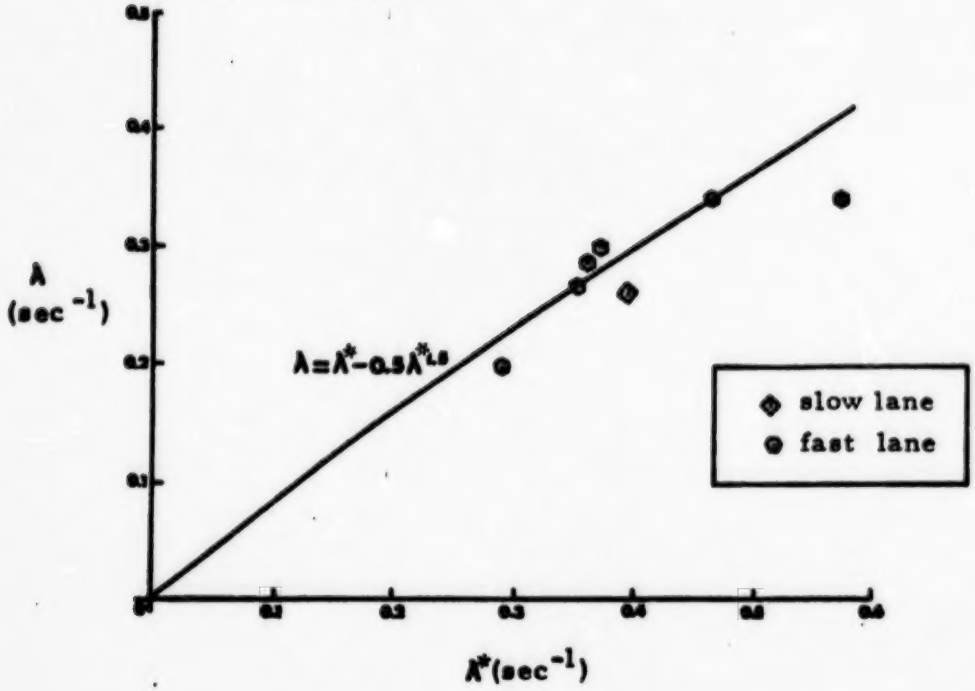
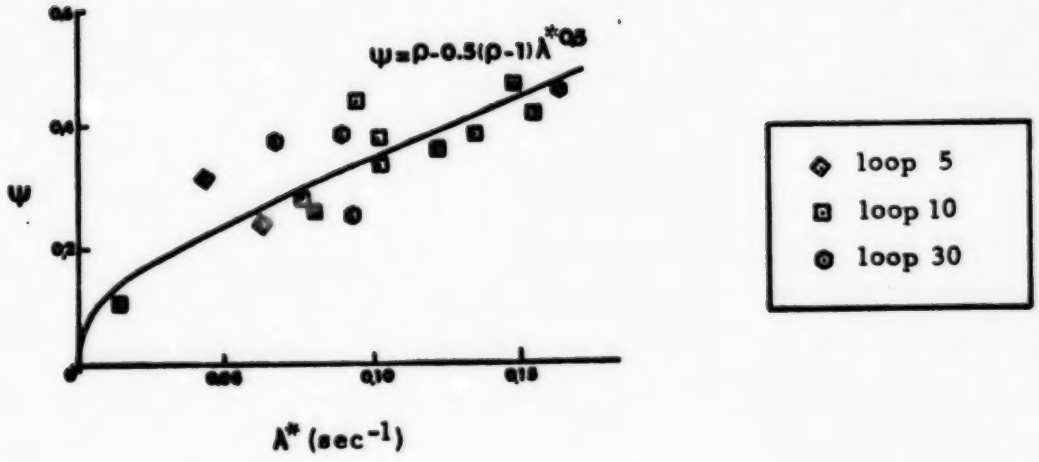


figure 10

INDIANA DATA



M4 FAST LANE DATA